Section 11.1: Sequences and Series

Video 1: Sequences

A **sequence** is a function that computes an ordered list.

A **finite sequence** has the set of natural numbers {1, 2, …, *n*} as its domain.

An **infinite sequence** has the set of natural numbers {1, 2, 3, …} as its domain.

We use a_n to represent the **general term** or **nth term**.

a_n

1) List the first 5 terms of the given sequence.

$f(n) = 5n + 4$

$D: \{1, 2, 3, 4, 5\}$

a) $a_n = 5n + 4$

$5(1) + 4 = 9$ $\quad 5(2) + 4 = 14$ $\quad 5(3) + 4 = 19$

$5(4) + 4 = 24$ $\quad 5(5) + 4 = 29$ $\qquad 9, 14, 19, 24, 29$

b) $a_n = \left(\frac{1}{2}\right)^n$

$(1/2)^1 \quad (1/2)^2 \quad (1/2)^3 \quad (1/2)^4 \quad (1/2)^5$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

Converges to 0

c) $a_n = n^2$

$1^2 \quad 2^2 \quad 3^2 \quad 4^2 \quad 5^2$

$1, 4, 9, 16, 25$

Diverges

b) $(-1)^n \cdot n^3$

$(-1)^{n+1}$

$(-1)^1 \cdot 1^3 \quad (-1)^2 \cdot 2^3 \quad (-1)^3 (3)^3 \quad (-1)^4 \cdot 4^3 \quad (-1)^5 \cdot 5^3$

$-1, 8, -27, 64, -125$

c) $a_n = \frac{2n+7}{n^2-3}$

$\frac{2(1)+7}{1^2-3} \quad \frac{2(2)+7}{2^2-3} \quad \frac{2(3)+7}{3^2-3} \quad \frac{2(4)+7}{4^2-3} \quad \frac{2(5)+7}{5^2-3}$

$-\frac{9}{2}, 11, \frac{13}{6}, \frac{15}{13}, \frac{17}{22}$

Video 2: Recursively Defined Sequences

Besides listing a formula for the general term of a sequence, we can define a sequence by listing its first term, a_1, and a formula for how to create any other term from the preceding term.

2) Find the first 5 terms of the given sequence.

a) $a_1 = 6$, $a_n = 4 \cdot a_{n-1} - 5$

$4\cdot6-5 \quad 4\cdot19-5 \quad 4\cdot71-5 \quad 4\cdot279-5$

$6, \quad 19, \quad 71, \quad 279, \quad 1111$

$$\begin{array}{r} 279 \\ \times\ 4 \\ \hline 1116 \\ -\ 5 \\ \hline 1111 \end{array}$$

b) $a_1 = -4$, $a_n = -2\left(a_{n-1}\right) + 4n - 5$

$a_1 = -4$

$n=2 \quad a_2 = -2(-4) + 4(2) - 5 = 11$

$n=3 \quad a_3 = -2(11) + 4(3) - 5 = -15$

$n=4 \quad a_4 = -2(-15) + 4(4) - 5 = 41$

$n=5 \quad a_5 = -2(41) + 4(5) - 5 = -67$

$-4, 11, -15, 41, -67$

Fibonacci Sequence

$a_1 = 1 \quad a_2 = 1 \quad$ for $n \geq 3$

$a_n = a_{n-1} + a_{n-2}$

$1, 1, 2, 3, 5, 8, 13, \ldots$

Video 3: Series and Summation Notation

Σ : sigma

A **series** is the sum of a sequence.

S_n **Finite series**: $S_n = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{n} a_i$. ← start at $i=1$ go through $i=n$ adding a_i

S_∞ **Infinite series**: $S_\infty = a_1 + a_2 + a_3 + \ldots = \sum_{i=1}^{\infty} a_i$.

3) Evaluate.

a) $\sum_{i=1}^{5}(3i+7)$ $= [3(1)+7] + [3(2)+7] + [3(3)+7] + [3(4)+7] + [3(5)+7]$

$= 10 + 13 + 16 + 19 + 22$

$= 80$

b) $\sum_{i=1}^{6}\left(i^2+3i-4\right)$

$1^2+3(1)-4 = 0$

$2^2+3(2)-4 = 6$

$3^2+3(3)-4 = 14$

$4^2+3(4)-4 = 24$

$5^2+3(5)-4 = 36$

$6^2+3(6)-4 = 50$

130

1 = 4^1
2
⋮
8

c) $\sum_{i=1}^{8} 4^i$

$= 4^1 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6 + 4^7 + 4^8$

$= 4 + 16 + 64 + 256 + 1024 + 4096 + 16,384 + 65,536$

$= 87,380$

4) Write the terms for the series, and evaluate (if possible).

a) $\sum_{i=6}^{10} a_i$ $= a_6 + a_7 + a_8 + a_9 + a_{10}$

b) $\sum_{i=1}^{4}(5x_i+8)$, if $x_1=6, x_2=10, x_3=14,$ and $x_4=18$

$[5(\underset{6}{x_1})+8] + [5(\underset{10}{x_2})+8] + [5(\underset{14}{x_3})+8] + [5(\underset{18}{x_4})+8]$

$= 38 + 58 + 78 + 98$

$= 272$

Δx : delta x

c) $\sum_{i=1}^{3} f(x_i)\cdot\Delta x$, if $f(x)=x^3, x_1=5, x_2=9, x_3=13,$ and $\Delta x=4$.

$f(x_1)\cdot\Delta x + f(x_2)\cdot\Delta x + f(x_3)\cdot\Delta x$

$x_1^3\cdot 4 + x_2^3\cdot 4 + x_3^3\cdot 4$

$= 5^3\cdot 4 + 9^3\cdot 4 + 13^3\cdot 4$

$= 125\cdot 4 + 729\cdot 4 + 2197\cdot 4$

$= 12,204$

Video 4: Summation Properties and Rules

Summation Properties	**Summation Rules**
$\sum_{i=1}^{n} c = n \cdot c$	$\sum_{i=1}^{n} i = 1+2+\ldots+n = \frac{n(n+1)}{2}$
$\sum_{i=1}^{n} c \cdot a_i = c \cdot \sum_{i=1}^{n} a_i$	$\sum_{i=1}^{n} i^2 = 1^2+2^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\sum_{i=1}^{n} i^3 = 1^3+2^3+\ldots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$
$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$	

5) Use the summation properties and rules to evaluate each series.

a) $\sum_{i=1}^{27} 8$

$$= \overbrace{8 + 8 + 8 \ldots + 8}^{27}$$

$$27 \cdot 8 = 216$$

b) $\sum_{i=1}^{15} 4i^3$

$$= 4 \cdot \sum_{i=1}^{15} i^3$$

$$= 4 \left(\frac{15 \cdot \cancel{16}^{8}}{\cancel{2}_{1}}\right)^2 \leftarrow$$

$$= 4 (120)^2$$

$$= 4 \cdot 14{,}400$$

$$= 57{,}600$$

$$\left(\frac{n(n+1)}{2}\right)^2$$

c) $\sum_{i=1}^{12}(9i-7)$ $= \sum_{i=1}^{12} 9i - \sum_{i=1}^{12} 7$

$= 9\left(\frac{n(n+1)}{2}\right) - 12 \cdot 7$

$= 9\left(\frac{\cancel{12}^{6} \cdot 13}{\cancel{2}_{1}}\right) - 12 \cdot 7$

$= 9 \cdot 78 - 12 \cdot 7$

$= 702 - 84 = 618$

d) $\sum_{i=1}^{7}(i^2+8i+12)$

$= \sum_{i=1}^{7} i^2 + 8\sum_{i=1}^{7} i + \sum_{i=1}^{7} 12$

$i^2: \frac{n(n+1)(2n+1)}{6}$

$i: \frac{n(n+1)}{2}$

$7+1$ 4 $2(7)+1$

$= \frac{7(\cancel{8})(\cancel{15})^{5}}{\cancel{6}\,\cancel{3}\,1} + 8 \cdot \frac{7 \cdot \cancel{8}^{4}}{\cancel{2}_{1}} + 7 \cdot 12$

$= 7 \cdot 4 \cdot 5 + 8 \cdot 7 \cdot 4 + 7 \cdot 12$

$= 140 + 224 + 84$

$= 448$